

CONTENTS.

Chapter 1. Introduction.	7
§ 1.1. Algorithms for diophantine equations.	7
§ 1.2. The Gelfond-Baker method.	15
§ 1.3. Theoretical diophantine approximation.	17
§ 1.4. Computational diophantine approximation.	19
§ 1.5. The procedure for reducing upper bounds.	26
Chapter 2. Preliminaries.	28
§ 2.1. Algebraic number theory.	28
§ 2.2. Some auxiliary results.	30
§ 2.3. p -adic numbers and functions.	31
§ 2.4. Lower bounds for linear forms in logarithms.	33
§ 2.5. Numerical methods.	36
Chapter 3. Algorithms for diophantine approximation.	40
§ 3.1. Introduction.	40
§ 3.2. Homogeneous one-dimensional approximation in the real case: continued fractions.	41
§ 3.3. Inhomogeneous one-dimensional approximation in the real case: the Davenport lemma.	43
§ 3.4. The L^3 -lattice basis reduction algorithm, theory.	45
§ 3.5. The L^3 -lattice basis reduction algorithm, practice.	49
§ 3.6. Finding all short lattice points: the Fincke and Pohst algorithm.	55
§ 3.7. Homogeneous multi-dimensional approximation in the real case: real approximation lattices.	57
§ 3.8. Inhomogeneous multi-dimensional approximation in the real case: an alternative for the generalized Davenport lemma.	60
§ 3.9. Inhomogeneous zero-dimensional approximation in the p -adic case.	64
§ 3.10. Homogeneous one-dimensional approximation in the p -adic case: p -adic continued fractions and approximation lattices of p -adic numbers.	66

§3.11.	Homogeneous multi-dimensional approximation in the p-adic case: p-adic approximation lattices.	67
§3.12.	Inhomogeneous one- and multi-dimensional approximation in the p-adic case.	69
§3.13.	Useful sublattices of p-adic approximation lattices.	71
Chapter 4.	S-integral elements of binary recurrence sequences.	74
§ 4.1.	Introduction.	74
§ 4.2.	Binary recurrence sequences.	76
§ 4.3.	The growth of the recurrence sequence.	78
§ 4.4.	Upper bounds.	83
§ 4.5.	Symmetric recurrences: an elementary method.	86
§ 4.6.	A basic lemma, and some trivial cases.	89
§ 4.7.	The reduction algorithm in the hyperbolic case.	91
§ 4.8.	The reduction algorithm in the elliptic case.	95
§ 4.9.	The generalized Ramanujan-Nagell equation.	98
§4.10.	A mixed quadratic-exponential equation.	102
Chapter 5.	The inequality $0 < x - y < y^\delta$ in S-integers.	105
§ 5.1.	Introduction.	105
§ 5.2.	Upper bounds for the solutions.	106
§ 5.3.	Reducing the upper bounds in the one-dimensional case.	107
§ 5.4.	Reducing the upper bounds in the multi-dimensional case.	109
§ 5.5.	Tables.	112
Chapter 6.	The equation $x + y = z$ in S-integers .	118
§ 6.1.	Introduction.	118
§ 6.2.	Upper bounds.	119
§ 6.3.	The p-adic approximation lattices.	121
§ 6.4.	Reducing the upper bounds in the one-dimensional case.	123
§ 6.5.	Reducing the upper bounds in the multi-dimensional case.	126
§ 6.6.	Examples related to the abc-conjecture.	128
§ 6.7.	Tables.	130
Chapter 7.	The sum of two S-units being a square.	139
§ 7.1.	Introduction.	139
§ 7.2.	The case $D = 1$.	140
§ 7.3.	Towards generalized recurrences.	141
§ 7.4.	Towards linear forms in logarithms.	145
§ 7.5.	Upper bounds for the solutions: outline.	150
§ 7.6.	Upper bounds for the solutions: details.	153
§ 7.7.	The reduction technique.	161

§ 7.8.	The standard example.	161
§ 7.9.	Tables.	170
Chapter 8.	The Thue equation.	181
§ 8.1.	Introduction.	181
§ 8.2.	From the Thue equation to a linear form in logarithms.	182
§ 8.3.	Upper bounds.	187
§ 8.4.	Reducing the upper bound.	191
§ 8.5.	An application: integral points on the elliptic curve $y^2 = x^3 - 4x + 1$.	195
§ 8.6.	The Thue-Mahler equation, an outline.	205
References.		207
Samenvatting.		215
Curriculum vitae.		219